

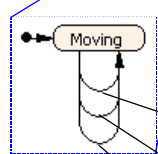
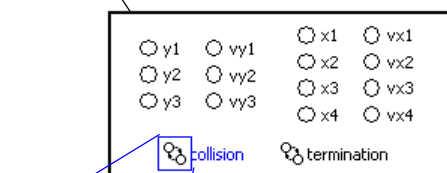
## C12 Spheres' Collision – AnyLogic

### Numerical simulation / Time-oriented Model

**Simulator:** AnyLogic ([www.xjtek.com](http://www.xjtek.com)) is a general-purpose simulation environment for discrete, continuous and hybrid systems. It employs UML-RT structure diagrams for building hierarchical models in object-oriented way and *hybrid statecharts* for behaviour specification. The generated model is Java and can be extended with user's Java code. The simulation engine handles discrete events and dynamically changing sets of algebraic-differential equations. It automatically detects "change" (or "state") events.

**Model:** The system of colliding spheres is modelled as one AnyLogic object with several variables representing the positions of spheres ( $x1, x2, x3, x4$ ), their relative distances ( $y1, y2, y3$ ), and their absolute ( $vx1, vx2, vx3, vx4$ ) and relative ( $vy1, vy2, vy3$ ) velocities. The equations for motion of the spheres are associated with the object. The statechart has a single state and three transitions representing collisions of spheres. These transitions are triggered by "change events" – Boolean expressions over variables.

$$\begin{aligned} y1 &= x2-x1-d & vx2 &= vx1+vy1 & d(x1)/dt &= vx1 \\ y2 &= x3-x2-d & vx3 &= vx2+vy2 & d(x2)/dt &= vx2 \\ y3 &= x4-x3-d & vx4 &= vx3+vy3 & d(x3)/dt &= vx3 \\ & & & & d(x4)/dt &= vx4 \end{aligned}$$

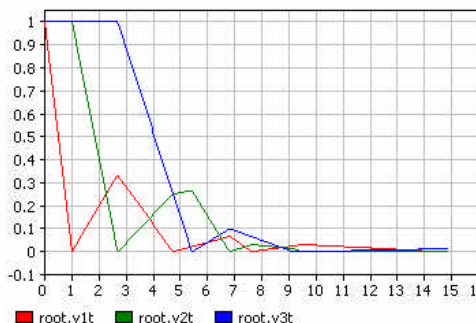


Trigger  
 $y1 < 0 \ \&\& \ vy1 < 0$   
 Action  
 $vx1 = vx1 + (1+e) * m2 / (m1+m2) * vy1;$   
 $vy2 = vy2 + (1+e) * m1 / (m1+m2) * vy1;$   
 $vy1 = -e * vy1;$

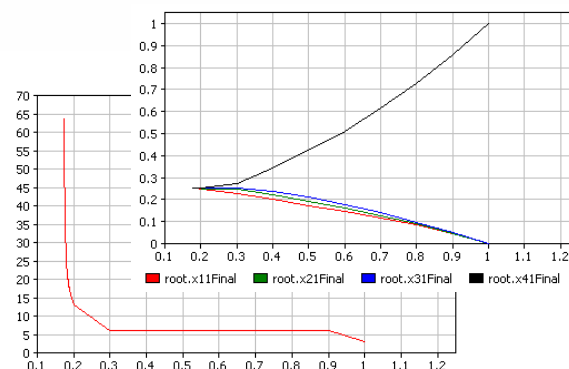
The hybrid statechart can be entered in AnyLogic model editor, and the simulation engine automatically detects change events, changes the working set of equations and readjusts the numerical methods.

**Task a: Simulation of the System. a1:** The following figure shows the distance-time functions  $y1(t)$ ,  $y2(t)$  and  $y3(t)$  for  $e = 0.2$ ,  $d = 1$  and  $a = 1$ ,  $v0 = 1$  in time  $0 \leq t \leq 15$  is shown on the diagram below.

**Task a2.** For the elastic case ( $e=1$ ) the final velocities of the spheres are exactly  $[0, 0, 0, 1]$ , for the quasi-plastic case ( $e = 0.172$ )  $[0.25, 0.25, 0.25, 0.25]$ .



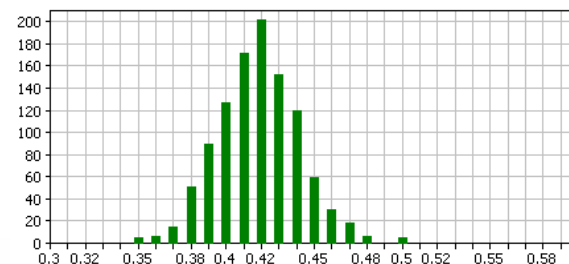
**Task b: Variation of restitution coefficient:** The number of collisions and final velocities as functions of the restitution coefficient presented in the picture below were obtained in a number of replications with varying parameter controlled by Java script.



**Task c1: Boundary value problem.** A binary search was implemented within AnyLogic environment as Java script. AnyLogic obtains the value of  $e = 0.5874011$  with the precision set to  $1e-7$ .

**Task c2: Stochastic deviation of restitution.** Using the built-in normal probability distribution for  $e$  AnyLogic has built the histogram shown below. The (built-in) statistical analysis gives the following results:

Mean	0.42317935397	Variance	4.54830611E-4
Min	0.35753660792	Deviation	0.02132675811
Max	0.50359726405	MeanConf	0.00132184616



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